

Dominic Orchard

Vilem-Benjamin Liepelt

Harley Eades III





https://granule-project.github.io/

# Data









#### (Some) data acts as a resource Ignoring this leads to bugs!

#### Solution

#### Capture resource constraints in types Do this in an extensible way





Precision

Track information for quantitative reasoning types Graded modal types

Indexed

SMT solver (e.g. Z3)

Be strict about resources

Linear types

# Why linearity?

- Efficiency
  - In-place update
- Usage protocols & resources
  - File handles\*
  - Streaming I/O
  - Session typed channels\*
  - Language interoperability
  - Low-level stateful protocols
  - Pointers/References\*\*
  - Mutable arrays\*\*
  - Circuit modeling
- Program reasoning

\*implemented in Granule

\*\*experimental support / work in progress

## **Initial Demo**

#### Linearity, modalities, and two graded modalities

# $\Box_n A \quad \text{where } n \in \mathbb{N}$ $\Box_{[n..m]} A \quad \text{where } n, m \in \mathbb{N} \land n \leq m$

### Graded modalities (informally)



### Graded necessity (coeffects/graded comonads)

weakening

- $A [m + n] \longrightarrow (A [m] \times A [n])$  contraction
- $A [1] \longrightarrow A$  use (dereliction)
- A  $[m * n] \rightarrow (A [m]) [n] composition (promotion)$
- $A[m] \rightarrow A[n]$  (where  $m \ge n$ ) approximation

#### Graded by pre-ordered semiring

 $A [0] \longrightarrow ()$ 

# Graded necessity in Granule

- (Extended) naturals
- Intervals
- Security levels
- Products
- (Extended) positive rationals
- (In progress) Monotonicity (à la Arntzenius & Krishnaswami)
- <insert your idea here>

# Dataflow paths



Grading semiring captures (backwards) dataflow

BLOG TALKS WORKSHOPS ABOUT OPEN INVITE PRACTICAL TLA+

#### THE GREAT THEOREM PROVER SHOWDOWN

⊡ \$7 \$7 ⊟ Apr 25, 2018

Functional programming and immutability are hot right now. On one hand, this is pretty great as there's lots of nice things about functional programming. On the other hand,

#### **The Questions**

 Leftpad. Takes a padding character, a string, and a total length, returns the string padded to that length with that character. If length is less than the length of the string, does nothing.

I don't like accepting things as axioms. If we make a claim, we better damn well put it to the test. So last Friday I did exactly that:



# Next demo

Verification via first-class grades and indexed types

# Next demo

#### **Security levels**

Public **Private** 4 Irrelevant

# Dataflow analysis via grading

Consider

$$\lambda x$$
 .  $\lambda y$  . let  $z = f x$  in  $(z, g z y)$ 

Flow graph

**Backward**  $\mathbf{h}_{i} = demands \text{ on } i^{th} \text{ parameter to } \mathbf{h}$ 







#### (monoid) graded possibility

# Potted history of graded types

2008 Durov - New approach to Arkelov geometry Smirnov - Graded monads and rings of polynomials



Key

- 2013 Petricek, O, Mycroft Coeffects: Unified Static Analysis of Context-Dependence
  2014 Ghica, Smith Bounded linear types in a resource semiring
  Brunel, Gaboardi, Mazza, Zdancewic A Core Quantitative Coeffect Calculus
  Katsumata Parametric effect monads and semantics of effect systems.
  O, Petricek, Mycroft The semantic marriage of effects and monads
  Petricek, O, Mycroft Coeffects: a calculus of context-dependent computation.
- 2016 Gaboardi, Katsumata, O, Breuvart, Uustalu Combining effects & coeffects via grading .... various coeffect papers

**2019** O, Liepelt, Eades - Quantitative program reasoning with graded modal types

# Come see Granule at ICFP 2019 https://granule-project.github.io/

 $J_{\text{CON}} = \overline{\Sigma \vdash A \sim A \triangleright \emptyset}$ 

Dominic Orchard, Vilem-Benjamin Liepelt, and Harley Eades III

1:14

$$\frac{\Sigma \vdash A' \sim A \rhd \theta_{1} \qquad \Sigma \vdash \theta_{1}B \sim \theta_{1}B' \rhd \theta_{2}}{\Sigma \vdash A \rightarrow B \sim A' \rightarrow B' \rhd \theta_{1} \uplus \theta_{2}} \qquad U_{\rightarrow} \qquad \frac{\Sigma \vdash A \sim A' \rhd \theta_{1} \qquad \Sigma \vdash \theta_{1}B \sim \theta_{1}B' \rhd \theta_{2}}{\Sigma \vdash A \Rightarrow B \sim A' \rightarrow B' \rhd \theta_{1} \uplus \theta_{2}} \qquad U_{\rightarrow} \qquad \frac{\Sigma \vdash A \Rightarrow A' \rhd \theta_{1} \ \exists \theta_{2}}{\Sigma \vdash A \Rightarrow A' \land \varphi} \qquad \frac{\Sigma \vdash A \Rightarrow A' \rhd \theta_{1} \ \exists \theta_{2}}{\Sigma \vdash \varphi \land \varphi} \qquad U_{APP}$$

Fig. 2. Ty

Type unification is given by relation  $\Sigma$ congruence over the structure of types (unc variables to types, e.g.  $(U_{\text{VAR}\exists})$  for  $\alpha \sim A$  ( here for brevity). Universally quantified va with unification variables via  $(U_{\text{VAR}\exists})$ . In n subterms are then applied to types being u extends to grading terms, which can also it is straightforward and follows a simila Substitutions can be typed by a type-10

#### Quantitative program reasoning with graded modal types DOMINIC ORCHARD, University of Kent, UK VILEM-BENJAMIN LIEPELT, University of Kent, UK HARLEY EADES III, Augusta University, USA

In programming, data is often considered to be infinitely copiable, arbitrarily discardable, and universally unconstrained. However this view is naïve: some data encapsulates resources that are subject to protocols (e.g., file and device handles, channels); some data should not be arbitrarily copied or communic

1